Quasinormal-mode description of waves in one-dimensional photonic crystals

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Quasinormal-mode treatment is extended to the description of scalar field behavior in one-dimensional photonic crystals. A one-dimensional photonic crystal is a particular configuration of an open cavity, where discontinuities of the refractive index give rise to field confinement. This paper presents, for a one-dimensional photonic crystal, a discussion about the completeness of the quasinormal-mode representation and, moreover, a discussion on the complex eigenfrequencies, as well as the corresponding field distribution. The concept of density of modes is also discussed in terms of quasinormal modes.

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I. INTRODUCTION

The definition of natural modes of confined structures is one of the central problems in physics [1,2], such as in nuclear physics, astrophysics, etc. The main problem is due to the boundary conditions, when they are such to push out the problem from the class of Sturm-Liouville problems. This occurs when boundary conditions imply the presence of eigenvalues, as for example when a scatterer excited from the outside [3] gives rise to a transmitted and reflected field.

An open cavity with external or internal excitation represents a "noncanonical" problem, in the sense of a Sturm-Liouville problem, due to the fact that the cavity modes couple themselves with the external modes. This problem is crucial when we intend to study light matter interaction effects, such as absorption, spontaneous emission, and stimulated emission, as they occur in microcavities.

The problem of the field description inside an open cavity has been discussed by several authors [4–6]. In particular, Leung *et al.* introduced description of the electromagnetic field in a one sided open optical cavity in terms of "quasinormal modes" or QNMs [7–10]. Because of the "leakage," or coupling between cavity modes and the continuum, the modes of the cavity are referred to as quasinormal modes and are characterized by complex frequencies. In Refs. [7–10], QNMs are discussed in a one-dimensional leaky cavity, provided the cavity is defined by a discontinuity in the refractive index which must approach its constant asymptotic value sufficiently rapidly.

In what follows, we extend the QNM treatment to one-dimensional photonic band gap structures. The past two decades have witnessed an intense investigation on electromagnetic propagation phenomena at optical frequencies in periodic structures, usually referred to as one-dimensional (1D) photonic band gap (1D-PBG) structures or 1D photonic crystals (1D PC) [11]. The essential properties of these structures are the existence of allowed and forbidden frequency bands and gaps, in analogy with energy bands and gaps in semiconductors. Dispersive properties are usually evaluated assuming an infinite periodic structure [12]. The finite dimensions of PCs conceptually modify the calculation and the nature of the dispersive properties: this is mainly due to the existence of an energy flow into and out of the crystal. A phenomenological approach to the dispersive properties of 1D PBG has been presented in Ref. [13]. Application of the effective-medium approach to a 1D PBG is discussed, and the analogy of a 1D PBG to a simple Fabry-Perot structure is developed by Sipe, Poladian, and Martijn de Sterke in Ref. [14].

Finite length photonic band gap structures manifest all the aspects related to a class of problems which do not belong to the Sturm-Liouville class: in fact, they behave as scatterer objects when they are excited from outside and as open cavities when excited from the inside.

In this work, we extend the QNM theory to 1D PBG as cavities open from both sides. The validity of the method is proved by reconstructing the field behavior inside the 1D PBG and by recovering the behavior of the density of modes.

The paper is organized as follows. In Sec. II, we discuss the completeness of the QNM representation for 1D-PBG structures. In Sec. III, we discuss QNM frequencies and functions. Finally, in Sec. IV we discuss the problem of the density of QNMs.

II. COMPLETENESS OF QNM REPRESENTATION FOR 1D-PBG STRUCTURES

Now, as depicted in Fig. 1(a), let us consider a 1D PBG as a cavity open at both ends, with refractive index that is continuous in some intervals

$$n(x)=\begin{cases} n_0(x) & \text{for} \quad x<x_0 \\ n_j(x) & \text{for} \quad x_{j-1}<x<x_j, \quad \text{where} \quad j\in[1,N]. \\ n_{N+1}(x) & \text{for} \quad x>x_N \end{cases} \tag{2.1}$$

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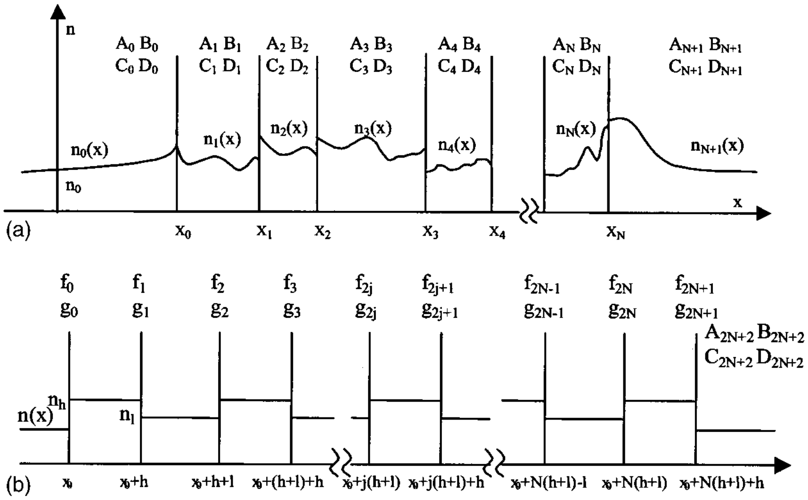

FIG. 1. (a) Typical behavior of the refractive index $n(x)$ in a 1D PBG. Index $n(x)$ is generally continuous with N discontinuities and it is asymptotic for large distance. (b) Refractive index $n(x)$ for a symmetric 1D PBG.

Outside the 1D-PBG structure there is a medium with an asymptotic refractive index

$$\lim_{x\to-\infty} n_0(x) = \lim_{x\to\infty} n_{N+1}(x) = n_0 . \tag{2.2}$$

According to the method proposed in Refs. [8–10], we use the Green function formalism [15]. The Fourier transform $\tilde{G}(x,y;\omega)$ of the Green function satisfies the following equation:

$$\left[\frac{\partial^2}{\partial x^2} + \omega^2 \rho(x)\right]\tilde{G}(x,y;\omega) = -\delta(x-y) \tag{2.3}$$

with

$$\rho(x) = \left(\frac{n(x)}{c}\right)^2 . \tag{2.4}$$

$\tilde{G}(x,y;\omega)$ is analytic when $\mathrm{Im}(\omega)>0$ and its behavior is of the type $\exp(\pm i n_0 \omega x/c)$ for $x\to\pm\infty$. Two auxiliary functions $g_\pm(x,\omega)$ are introduced, and solution of the homogeneous equation

$$\left[\frac{\partial^2}{\partial x^2} + \omega^2 \rho(x)\right] g_\pm(x,\omega) = 0, \tag{2.5}$$

with the asymptotic conditions

$$g_+(x,\omega) = e^{i n_0 (\omega/c) x} \quad \text{for} \quad x\to+\infty, \tag{2.6}$$

$$g_-(x,\omega) = e^{-i n_0 (\omega/c) x} \quad \text{for} \quad x\to-\infty. \tag{2.7}$$

The Wronskian associated to the two homogeneous equations for $g_\pm(x,\omega)$ is x independent [9]

$$W(\omega) = g_+(x,\omega) g'_-(x,\omega) - g_-(x,\omega) g'_+(x,\omega). \tag{2.8}$$

It can be shown that

$$\tilde{G}(x,y;\omega) = \begin{cases} -\dfrac{g_-(x,\omega) g_+(y,\omega)}{W(\omega)} & \text{for} \quad 0<x<y \\[2ex] -\dfrac{g_+(x,\omega) g_-(y,\omega)}{W(\omega)} & \text{for} \quad 0<y<x. \end{cases} \tag{2.9}$$

The QNMs correspond to all the poles of function $\tilde{G}(x,y;\omega)$; they represent couples of values $[\omega_n, f_n(x)]$; if a complex frequency is chosen so as to correspond to a QNM frequency $\omega=\omega_n$, it follows that [9]

$$W(\omega_n) = 0, \tag{2.10}$$

the auxiliary functions $g_\pm(x,\omega_n)$ are not linearly independent

$$f_n(x) = g_+(x,\omega_n) = c(\omega_n) g_-(x,\omega_n), \tag{2.11}$$

where $c(\omega_n)$ is an appropriate constant of proportionality, and satisfy the asymptotic conditions

$$f_n(x) = \exp(\pm i n_0 \omega_n x/c) \quad \text{for} \quad x\to\pm\infty. \tag{2.12}$$

The QNM treatment presents formal analogies with the treatment of Hermitian systems [8]. The most conspicuous similarity is the form of the solutions of wave equation, given as $E_n(x,t) = f_n(x) e^{-i\omega_n i}$. Frequencies ω_n become complex, with $\mathrm{Im}(\omega_n)<0$, and it is evident that the modes $E_n(x,t)$ are not stationary.

It is possible to extend the concept of norm [9] for a cavity open at both ends, of length L, from $x=0$ to $x=L$, i.e.,

$$\langle f_n|f_n\rangle = 2\omega_n \int_0^L \rho(x) f_n^2(x)\,dx + i\sqrt{\rho_0}[f_n^2(0)+f_n^2(L)]. \tag{2.13}$$

Several remarks about this generalized norm are in order: it involves $f_n^2(x)$ rather than $|f_n(x)|^2$ and it is in general complex; it involves two "surface terms" $i\sqrt{\rho_0}f_n^2(0)$ and $i\sqrt{\rho_0}f_n^2(L)$.

The representation of quasinormal modes is complete only inside the cavity [9]. We prove in what follows that, inside a 1D-PBG structure, the condition for QNM completeness is valid, i.e., the behavior of $\tilde{G}(x,y;\omega)$ for large $|\omega|$ is (see Ref. [9])

$$\lim_{|\omega|\to\infty} \tilde{G}(x,y;\omega) = 0, \quad \forall \quad \omega/\mathrm{Im}(\omega)<0. \tag{2.14}$$

The proof of QNM completeness is based on the application of the WKB method extended to optical regime. The WKB method, proposed to solve the Schrödinger equation (applied to $\hbar$ parameter, considering $\hbar\to 0$), here is applied in optics and for the λ parameter (considering $\lambda\to 0$, see Appendix A) [9]. We note that for a 1D PBG with refractive index (2.1), depicted in Fig. 1, we cannot solve exactly Eq. (2.5) with the asymptotic conditions (2.6) and (2.7); however, we can use a WKB-like method [7] in every period of the 1D PBG, if we suppose that λ is so small to verify

$$\left|\frac{dn_j(x)}{dx}\right| \ll \frac{4\pi}{\lambda} \quad \text{for} \quad x_{j-1}<x<x_j, \quad \text{where} \quad j\in[1,N], \tag{2.15}$$

where λ is the wavelength of the electromagnetic field.

For a 1D-PBG structure, whose refractive index is given by Eq. (2.1), we obtain the following expressions for the auxiliary function $g_-(x,\omega)$:

$$g_-(x,\omega) = A_j(\omega)\exp\left[i\frac{\omega}{c}\int_x^{x_j} n(\xi)d\xi\right] + B_j(\omega) \times\exp\left[-i\frac{\omega}{c}\int_x^{x_j} n(\xi)d\xi\right], \quad x_{j-1}<x<x_j$$

$$g_-(x,\omega) = \exp\left[-i\frac{\omega}{c}\int_x^{x_0} n(\xi)d\xi\right], \quad x<x_0, \tag{2.16}$$

where $j\in[1,N+1]$ and $x_{N+1}=+\infty$. For the auxiliary function $g_+(x,\omega)$, we have

$$g_+(x,\omega) = C_j(\omega)\exp\left[i\frac{\omega}{c}\int_{x_{j-1}}^{x} n(\xi)d\xi\right] + D_j(\omega) \times\exp\left[-i\frac{\omega}{c}\int_{x_{j-1}}^{x} n(\xi)d\xi\right], \quad x_{j-1}<x<x_j$$

$$g_+(x,\omega) = \exp\left[i\frac{\omega}{c}\int_{x_N}^{x} n(\xi)d\xi\right], \quad x>x_N, \tag{2.17}$$

where $j\in[0,N]$ and $x_{-1}=-\infty$.

Under the conditions of continuity for the auxiliary functions $g_\pm(x,\omega)$ and their derivatives in $x=x_j$, we obtain

$$\begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} = S_j \begin{pmatrix} A_j e^{-i\vartheta_j} + R_j B_j e^{-i\vartheta_j} \\ R_j A_j e^{i\vartheta_j} + B_j e^{i\vartheta_j} \end{pmatrix}, \tag{2.18}$$

$$\begin{pmatrix} C_j \\ D_j \end{pmatrix} = S_j' \begin{pmatrix} C_{j+1} e^{-i\vartheta_{j-1}} - R_j D_{j+1} e^{-i\vartheta_{j-1}} \\ -R_j C_{j+1} e^{i\vartheta_{j-1}} + D_{j+1} e^{i\vartheta_{j-1}} \end{pmatrix}, \tag{2.19}$$

where

$$\vartheta_j = \frac{\omega}{c}\int_{x_j}^{x_{j-1}} n(x)dx,$$

$$R_j = \frac{[n(x_j^+) - n(x_j^-)]}{[n(x_j^+) + n(x_j^-)]},$$

$$S_j = \frac{[n(x_j^+) + n(x_j^-)]}{2n(x_j^+)},$$

$$S_j' = \frac{[n(x_j^+) + n(x_j^-)]}{2n(x_j^-)}. \tag{2.20}$$

Now, only inside the 1D PBG, i.e., $\forall\ (x,y)|x_0<y\leq x<x_N$, we can assure that $\exists j|x_0<y\leq x_j$, $x_{j-1}\leq x<x_N$, so we have for $j\leq m\leq N$ and $1\leq n\leq j$,

$$g_-(y,\omega) = A_n(\omega)\exp\left[i\frac{\omega}{c}\int_y^{x_n} n(\xi)d\xi\right] + B_n(\omega) \times\exp\left[-i\frac{\omega}{c}\int_y^{x_n} n(\xi)d\xi\right] \tag{2.21}$$

$$g_+(x,\omega) = C_m(\omega)\exp\left[i\frac{\omega}{c}\int_{x_{m-1}}^{x} n(\xi)d\xi\right] + D_m(\omega) \times\exp\left[-i\frac{\omega}{c}\int_{x_{m-1}}^{x} n(\xi)d\xi\right]. \tag{2.22}$$

Then, the Fourier transform of the Green function for $y\leq x$ has the following expression:

$$\tilde{G}(x,y;\omega)=-\frac{\left[A_n(\omega)\exp\left[i\frac{\omega}{c}\int_y^{x_n}n(\xi)d\xi\right]+B_n(\omega)\exp\left[-i\frac{\omega}{c}\int_y^{x_n}n(\xi)d\xi\right]\right]\left[C_m(\omega)\exp\left[i\frac{\omega}{c}\int_{x_{m-1}}^{x}n(\xi)d\xi\right]+D_m(\omega)\exp\left[-i\frac{\omega}{c}\int_{x_{m-1}}^{x}n(\xi)d\xi\right]\right]}{2i\frac{\omega}{c}n(x)\left[D_m(\omega)B_n(\omega)\exp\left[-i\frac{\omega}{c}\int_{x_{m-1}}^{x_n}n(\xi)d\xi\right]-C_m(\omega)A_n(\omega)\exp\left[i\frac{\omega}{c}\int_{x_{m-1}}^{x_n}n(\xi)d\xi\right]\right]}. \tag{2.23}$$

Coefficients A_n, B_n, C_m, and D_m are obtained from Eqs. (2.18) and (2.19) after some algebra

$$\begin{pmatrix}A_n\\B_n\end{pmatrix}=\prod_{j=0}^{n-1}\begin{pmatrix}A_j\\B_j\end{pmatrix}\cong\prod_{j=0}^{n-1}S_j\begin{pmatrix}R_n\exp\left(i\sum_{j=0}^{n-2}\vartheta_j-\vartheta_{n-1}\right)\\ \exp\left(i\sum_{j=0}^{n-1}\vartheta_j\right)\end{pmatrix}, \tag{2.24}$$

$$\begin{pmatrix}C_m\\D_m\end{pmatrix}=\prod_{j=m}^{N}\begin{pmatrix}C_j\\D_j\end{pmatrix}\cong\prod_{j=m}^{N}S'_j\begin{pmatrix}R_NR_m\exp\left(i\sum_{j=m}^{N-1}\vartheta_j-\vartheta_{m-1}\right)\\ -R_N\exp\left(i\sum_{j=m-1}^{N-1}\vartheta_j\right)\end{pmatrix}. \tag{2.25}$$

We suppose for the refractive index that $|n(x_j^+)-n(x_j^-)|\leqslant\Delta n\leqslant 1$, $\forall\ j\in[1,N]$ so $R_j\leqslant\Delta n/[n(x_j^+)+n(x_j^-)]\leqslant 1/[n(x_j^+)+n(x_j^-)]<1/2$, $\forall\ j\in[1,N]$. We note that R_j, $\forall\ j\in[1,N]$, is close to 0 than to 1; then, from Eqs. (2.24) and (2.25), it follows that B_n is dominant with respect to A_n and D_m is dominant with respect to C_m, so Eq. (2.23) becomes

$$\tilde{G}(x,y,\omega)\cong-\frac{\exp\left(-i\frac{\omega}{c}\left[\int_y^{x_n}n(\xi)d\xi+\int_{x_{m-1}}^{x}n(\xi)d\xi\right]\right)}{2i\frac{\omega}{c}n(x)\exp\left[-i\frac{\omega}{c}\int_{x_{n-1}}^{x_n}n(\xi)d\xi\right]}. \tag{2.26}$$

Since for $y\leqslant x$ we have

$$\int_y^{x_n}n(\xi)d\xi+\int_{x_{m-1}}^{x}n(\xi)d\xi=\int_{x_{m-1}}^{x_n}n(\xi)d\xi+\int_y^{x}n(\xi)d\xi\leqslant\int_{x_{n-1}}^{x_n}n(\xi)d\xi \tag{2.27}$$

and the transformed Green function in a 1D-PBG structure has the following behavior:

$$\tilde{G}(x,y,\omega)\rightarrow 0\quad\text{for }|\omega|\rightarrow\infty. \tag{2.28}$$

Therefore, the QNM completeness in 1D PBG is proved.

III. QNM FREQUENCIES AND FUNCTIONS FOR PBG STRUCTURES

Let us now specify the previous considerations to a symmetric 1D PBG with N periods, where a period consists of two layers with refractive indices n_k and n_l, as usually considered in the literature. We divide the entire x space into $2N+3$ layers, in each of which the index of refraction $n(x)$ is constant; for a generic interval (x_{j-1},x_j), with $j=0,1,...,2N+1$, $2N+2$, and $x_{-1}=-\infty$ and $x_{2N+1}=\infty$, the index of refraction is chosen to take the constant values

$$n_j=\begin{cases}1 & \text{for}\quad j=0,\ 2N+2\\ n_h & \text{for}\quad j=1,3,...,2N-1,\ 2N+1\\ n_l & \text{for}\quad j=2,4,...,2N,\end{cases} \tag{3.1}$$

as schematically depicted in Fig. 1(b).

In Appendix B, we demonstrate that if we introduce the two phase terms

$$\delta_l=q_l l=n_l l\frac{\omega}{c},$$

$$\delta_h=q_h h=n_h h\frac{\omega}{c}, \tag{3.2}$$

the QNM frequencies can be found by solving the following transcendental equation:

$$\alpha\sum_{j=0}^{[(N-1)/2]}\frac{(-1)^j}{j!}\frac{(N-1-j)!}{(N-1-2j)!}(\gamma)^{N-1-2j}+\beta\sum_{j=0}^{[(N-2)/2]}\frac{(-1)^j}{j!}\frac{(N-2-j)!}{(N-2-2j)!}(\gamma)^{N-2-2j}=0, \tag{3.3}$$

where the coefficients α, β, and γ are parameters related to the refractive indices of each layer

$$\alpha=\frac{1}{4}\left\{\left[\left(n_1+\frac{1}{n_1}\right)+2\left(n_h+\frac{1}{n_h}\right)-4-2\left(\frac{n_h}{n_1}+\frac{n_l}{n_h}\right)\right.\right.$$
$$+\left.\left(\frac{n_h^2}{n_1}+\frac{n_1}{n_h^2}\right)\right]e^{2i\delta_h+i\delta_l}+\left[\left(n_1+\frac{1}{n_1}\right)-2\left(n_h+\frac{1}{n_h}\right)-4\right.$$
$$+\left.2\left(\frac{n_h}{n_1}+\frac{n_1}{n_h}\right)+\left(\frac{n_h^2}{n_1}+\frac{n_1}{n_h^2}\right)\right]e^{-2i\delta_h+i\delta_l}+\left[2\left(n_1+\frac{1}{n_1}\right)\right.$$
$$-\left.2\left(\frac{n_h^2}{n_1}+\frac{n_1}{n_h^2}\right)\right]e^{i\delta_l}+\left[-\left(n_1+\frac{1}{n_1}\right)+2\left(n_h+\frac{1}{n_h}\right)-4\right.$$
$$+\left.2\left(\frac{n_h}{n_1}+\frac{n_1}{n_h}\right)-\left(\frac{n_h^2}{n_1}+\frac{n_1}{n_h^2}\right)\right]e^{2i\delta_h-i\delta_l}+\left[-\left(n_1+\frac{1}{n_1}\right)\right.$$
$$-\left.2\left(n_h+\frac{1}{n_h}\right)-4-2\left(\frac{n_h}{n_1}+\frac{n_1}{n_h}\right)-\left(\frac{n_h^2}{n_1}+\frac{n_1}{n_h^2}\right)\right]e^{-2i\delta_h-i\delta_l}$$
$$+\left.\left[-2\left(n_1+\frac{1}{n_1}\right)+2\left(\frac{n_h^2}{n_1}+\frac{n_1}{n_h^2}\right)\right]e^{-i\delta_l}\right\},$$

$$\beta=\left\{\left[2-\left(n_h+\frac{1}{n_h}\right)\right]e^{i\delta_h}+\left[2+\left(n_h+\frac{1}{n_h}\right)\right]e^{-i\delta_h}\right\},$$

$$\gamma=\frac{1}{4n_hn_1}\{(n_h+n_1)^2e^{i(\delta_h+\delta_l)}-(n_h-n_1)^2e^{i(\delta_h-\delta_l)}$$
$$-(n_h-n_1)^2e^{i(\delta_l-\delta_h)}+(n_h+n_1)^2e^{-i(\delta_h+\delta_l)}\}. \quad (3.4)$$

More details are given in Appendix B. Only for a symmetric 1D PBG with quarter-wave stacks, Eq. (3.3) can be solved analytically; if N is the number of periods and $\omega_{\rm ref}$ is the reference frequency, there are exactly $2N+1$ QNM frequencies in the $[0,2\omega_{\rm ref}]$ range.

The QNMs distribution is not uniform in space, but presents gap structures. In Fig. 2(a), we plot QNM frequencies for a symmetric quarter-wave 1D PBG, where the reference wavelength is $\lambda_{\rm ref}=1\ \mu$m, the number of periods is $N=4$, and the two used refractive indices are $n_h=1.5$, $n_1=1$. A simple inspection of Fig. 2(a) shows that next to the gap, the QNM frequencies have the smallest imaginary part, and hence have the narrowest resonance lines. In Fig. 2(b), QNM frequencies are shown for the same structure of Fig. 2(a), but with $n_h=2$, $n_1=1$. Contrasting Figs. 2(a) and 2(b), it can be seen that as the difference between the refractive indices of adjacent layers is increased, the width of the gap increases. This also entails that the magnitude of the imaginary part of the QNM decreases, and the resonance peaks become tighter. In Fig. 2(c), QNM frequencies are shown for the same structure as above, with an increased number of periods $N=8$ and $n_h=2$, $n_1=1$. Contrasting Figs. 2(b) and 2(c), it can be seen that as the number of periods is increased, the position of the gap remains the same: as in the previous case, the imaginary parts of the QNMs decrease (in modulus), and resonance peaks become narrower.

If we study the behavior of the transmission spectrum of a symmetric quarter-wave 1D PBG, we observe that, in the base period $\omega\in[0,2\omega_{\rm ref}]$, the number of peaks is equal to $2N$. From the QNM theory (see Appendix B), in the range $\omega\in[0,2\omega_{\rm ref}]$, there are exactly $2N+1$ QNM frequencies. One of them is located on the imaginary axis and we reject it because it does not represent a physical field oscillation. The remaining $2N$ QNMs correspond to the $2N$ transmission peaks. The nth QNM frequency ω_n well describes the nth transmission peak in the sense that (1) $\mathrm{Re}(\omega_n)$ corresponds to the resonance frequency of the nth transmission peak and (2)

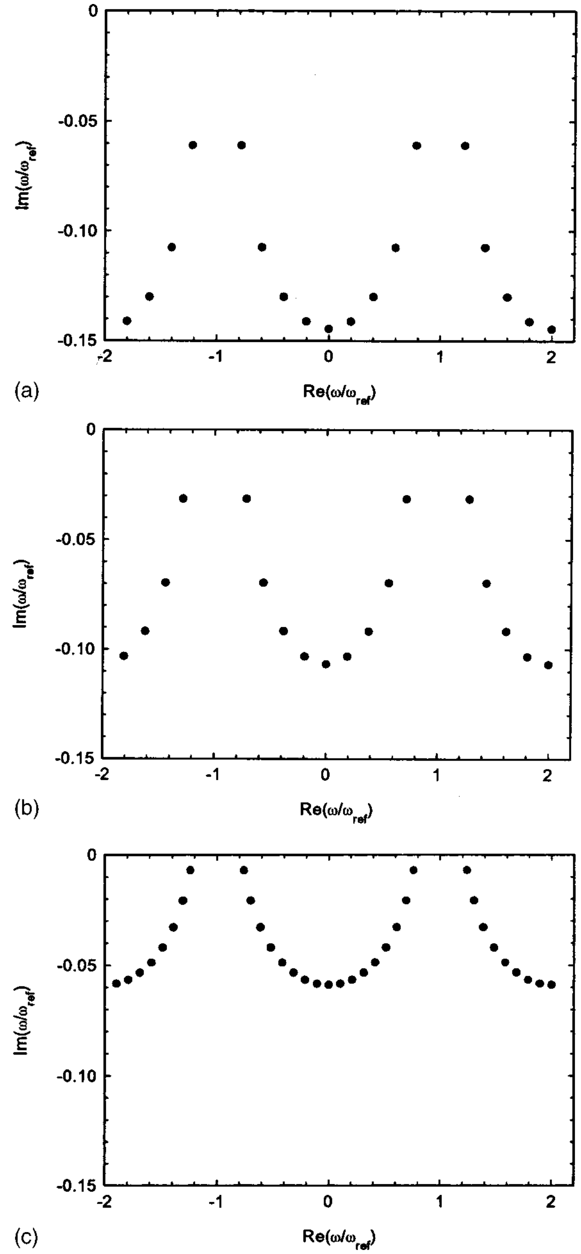

FIG. 2. (a) QNM frequencies for a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{\rm ref}=1\ \mu$m, number of periods $N=4$, and refractive indices $n_h=1.5$, $n_1=1$. (b) QNM frequencies for a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{\rm ref}=1\ \mu$m, number of periods $N=4$, and refractive indices $n_h=2$, $n_1=1$. (c) QNM frequencies for a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{\rm ref}=1\ \mu$m, number of periods $N=8$, and refractive indices $n_h=2$, $n_1=1$.

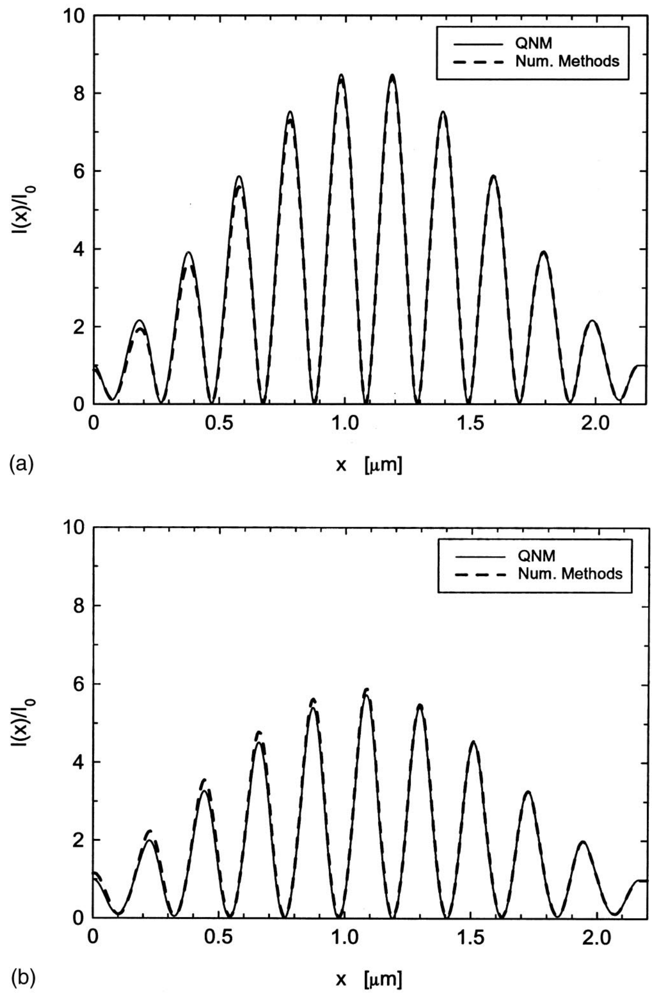

FIG. 3. (a) Intensity field distribution $I(x)$ normalized to input intensity I_0 for the QNM theory (——) and numerical methods (— — —), in a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{ref}=1\ \mu$m, number of periods $N=10$, and refractive indices $n_h=3$, $n_1=2$, at the low-wavelength band edge $\lambda_{low}=0.8688\ \mu$m. (b) Intensity field distribution $I(x)$ normalized to input intensity I_0, for the QNM theory (——) and numerical methods (— — —), in a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{ref}=1\ \mu$m, number of periods $N=10$, and refractive indices $n_h=3$, $n_1=2$, at the high-wavelength band edge $\lambda_{high}=1.1779\ \mu$m.

$|\mathrm{Im}(\omega_n)|$ is linked to the full width at half maximum of the nth transmission peak.

It is important to point out that (see Appendix B) the eigenfunctions $f_n(z)$ associated to the nth QNM, of eigenfrequency ω_n, is such that its square modulus $|f_n(z)|^2$ gives the field intensity distribution inside the PBG structure at frequency $\mathrm{Re}(\omega_n)$.

We consider a symmetric (N periods plus one stack) quarter-wave 1D-PBG structure with $N=10$, $\lambda_{ref}=1\ \mu$m, and $n_h=3$, $n_1=2$. The low-wavelength and high-wavelength band edges predicted by the QNM theory [Eq. (3.3)] are respectively 0.8688 μm and 1.1779 μm, those obtained by the numerical transfer matrix methods [16–18] are respectively 0.8677 μm and 1.1781 μm. In Fig. 3(a), we plot the field intensity distribution inside the same 1D-PBG structure as above, at the low-wavelength band edge (0.8688 μm), while, in Fig. 3(b), we refer to the high-wavelength band edge (1.1779 μm). We note that the field intensity distribution predicted by the QNM theory (see Appendix B) a part from negligible differences due to purely computational problems, is very close to that obtained by numerical methods based on the transmission matrix [16–18].

IV. DENSITY OF QUASINORMAL MODES

One of the most important parameters describing the spectral properties of field localization is the so called "density of modes." From the literature, the concept of density of modes for a closed cavity is well known, however the problem of a suitable definition arises for open cavities.

We would like to remark that the density of modes is calculated assuming the presence of an excitation at the left boundary of the 1D PBG.

For 1D-PBG structures, Bendickson, Dowling, and Scalora [18] introduced a definition of density of modes and a method of calculation based on the transfer matrix method. According to Ref. [18], the density of modes of a multilayer structures can be obtained from the phase of the complex transmission function

$$t(\omega)=x(\omega)+iy(\omega)=\sqrt{T}e^{i\phi}, \tag{4.1}$$

where φ is the total phase accumulated as the light propagates through the 1D PBG, seen as a potential well of width L. Therefore, the effect of propagation can be encapsulated into a propagation factor kL, where k is an "effective" wave number.

The quantity $dk/d\omega$ has the meaning of density of modes (DOM), and according to Eq. (4.1) it is given by

$$\sigma(\omega)=\frac{dk}{d\omega}=\frac{1}{L}\frac{y'x-x'y}{x^2+y^2}, \tag{4.2}$$

where the prime denotes differentiation with respect to ω.

Now, let us go back to the QNM formalism for open cavities and search for a definition and calculation of density of quasinormal modes, to be compared with results obtained from Eq. (4.2).

We deal with an open cavity, however through the concept of QNM we can "look" inside the cavity and define the local density of quasinormal modes $\sigma_{loc}(x,\omega)$ so that the number of QNMs $\delta N_{QNM}(x,\omega)$ in the infinitesimal tract of cavity $(x,x+dx)$ and for a range of frequency $(\omega,\omega+d\omega)$ is

$$\delta N_{QNM}(x,\omega)=\sigma_{loc}(x,\omega)dx\,d\omega. \tag{4.3}$$

For a cavity of length L and with a suitable refractive index $n(x)$, the density of quasinormal mode (DOM-QNM) is

$$\sigma(\omega)=\frac{1}{L}\int_0^L n^2(x)\sigma_{loc}(x,\omega)dx. \tag{4.4}$$

If we extend the method of Ho *et al.* [19] to a cavity open from both sides with a pump incoming from the left, we obtain the local density of quasinormal modes inside the structure as a superposition of QNMs

$$\sigma_{\mathrm{loc}}(x,\omega)=K\frac{\rho_0}{\pi}\sum_{n,m}\frac{F_n(0)F_m(0)}{(\omega-\omega_n)(\omega+\omega_m)}F_n(x)F_m(x), \tag{4.5}$$

where we have introduced a normalization constant K and the normalized QNM functions $F_n(x)=f_n(x)\sqrt{2\omega_n/\langle f_n,f_n\rangle}$.

For one resonance $m=-n$, since $(\omega_n^*,f_n^*)=(-\omega_{-n},f_{-n})$, the local density (4.5) becomes

$$\sigma_{\mathrm{loc},n}(x,\omega)=K\frac{\rho_0}{\pi}\frac{|F_n(0)F_m(x)|^2}{(\omega-\mathrm{Re}\,\omega_n)^2+\mathrm{Im}^2\,\omega_n}. \tag{4.6}$$

If we integrate Eq. (4.6) over the cavity length, we obtain

$$\sigma_n(\omega)=\frac{K}{\pi}\frac{l_n|F_n(0)|^2}{(\omega-\mathrm{Re}\,\omega_n)^2+\mathrm{Im}^2\,\omega_n}, \tag{4.7}$$

where we have introduced the normalization integrals $l_n=1/L\int_0^L F_n(x)|^2\rho(x)dx$.

Now, we proceed further and obtain an expression for the DOM-QNM of a symmetric quarter-wave 1D PBG.

For narrow resonances, i.e., $|\mathrm{Im}\,\omega_n|\ll|\mathrm{Re}\,\omega_n|$, we can neglect aliasing between frequencies, then the QNM-DOM is the superposition

$$\sigma(\omega)=\sum_n\sigma_n(\omega)=\frac{K}{\pi}\sum_n I_n\frac{|F_n(0)|^2}{(\omega-\mathrm{Re}\,\omega_n)^2+\mathrm{Im}^2\,\omega_n}. \tag{4.8}$$

As suggested in Ref. [19], we can calculate the normalization integrals $I_n=1/L\sqrt{\rho_0}/2|\mathrm{Im}\,\omega_n|[|F_n(0)|^2+|F_n(L)|^2]$. If the cavity is symmetric, then $F_n(L)=(-1)^nF_n(0)$; so taking into account that $|F_n(0)|^2=L/\sqrt{\rho_0}I_n|\mathrm{Im}\,\omega_n|$, Eq. (4.8) becomes

$$\sigma(\omega)=\frac{K'}{\pi}\sum_n I_n^2\frac{|\mathrm{Im}\,\omega_n|}{(\omega-\mathrm{Re}\,\omega_n)^2+\mathrm{Im}^2\,\omega_n}, \tag{4.9}$$

where $K'=KL/\sqrt{\rho_0}$.

As suggested in Ref. [19], under the previous hypothesis of $|\mathrm{Im}\,\omega_n|\ll|\mathrm{Re}\,\omega_n|$, we can approximate the normalization integrals to $I_n\cong 1/L$, and the DOM-QNM finally results

$$\sigma(\omega)=\frac{K_\sigma}{\pi}\sum_n\frac{|\mathrm{Im}\,\omega_n|}{(\omega-\mathrm{Re}\,\omega_n)^2+\mathrm{Im}^2\,\omega_n}, \tag{4.10}$$

where $K_\sigma=K'/L^2$.

Now, we specify, for a symmetric quarter-wave 1D PBG, the DOM-QNM (4.10). We consider a symmetric quarter-wave 1D PBG with a number of periods N and reference frequency ω_{ref}. The families of QNMs are $2N+1$. If we pick the m^{th} family, all the QNM frequencies have the same imaginary part $\mathrm{Im}(\omega_m)<0$. For any mth family, the QNM frequencies $\omega_{m,k}$ are distributed with a step $\Delta=2\omega_{\mathrm{ref}}$, so $\mathrm{Re}(\omega_{m,k})=\mathrm{Re}(\omega_{m,0})+k\Delta$, $k\in Z$ (see Appendix B). Then, the DOM-QNM (4.10) becomes

$$\sigma(\omega)=\frac{K_\sigma}{\pi}\sum_{m=0}^{2N}\sum_{k=-\infty}^{\infty}\frac{|\mathrm{Im}\,\omega_m|}{[\omega-(\mathrm{Re}\,\omega_{m,0}+k\Delta)]^2+\mathrm{Im}^2\,\omega_m}, \tag{4.11}$$

and it is a superposition of functions with QNM frequencies as parameters.

From above, there are $2N+1$ QNM frequencies in the $[0,2\omega_{\mathrm{ref}}]$ range, and, in conformity with above-mentioned notation, we express them as $\omega_{m,0}=\mathrm{Re}(\omega_{m,0})+j\,\mathrm{Im}(\omega_m)$, with $m\in[0,2N]$. Equation (4.11) converges to

$$\sigma(\omega)=\frac{K_\sigma}{2\Delta}\sum_{m=0}^{2N}\coth\left[j\frac{\pi}{\Delta}(\omega-\omega_{m,0}^*)\right]+j\frac{K_\sigma}{2\Delta}\sum_{m=0}^{2N}\cot\left[\frac{\pi}{\Delta}(\omega-\omega_{m,0})\right]. \tag{4.12}$$

For a symmetric quarter-wave 1D PBG with length L, number of periods N and reference frequency ω_{ref}, and the number of QNMs in units of L is $2N+1$ over the range $[0,2\omega_{\mathrm{ref}}]$ and represents also the QNMs in the whole range $[-2\omega_{\mathrm{ref}},2\omega_{\mathrm{ref}}]$, since the QNM of frequency $\omega_{m,0}$, with $\mathrm{Re}(\omega_{m,0})>0$, is represented also by the frequency $\omega_{-m,0}=-\omega_{m,0}^*$, with $\mathrm{Re}(\omega_{-m,0})<0$ [7]. Then, the normalization constant K_σ is obtained by following condition:

$$\int_{-2\omega_{\mathrm{ref}}}^{2\varphi_{\mathrm{ref}}}\sigma(\omega)d\omega=\frac{2N+1}{L}. \tag{4.13}$$

In Fig. 4(a), we plot the DOM predicted by the QNM theory [Eq. (4.12)] and by Bendickson [Eq. (4.2)] for a symmetric quarter-wave 1D PBG, where the reference wavelength is $\lambda_{\mathrm{ref}}=1\ \mu$m, the number of periods is $N=4$ and the two used refractive indices are $n_h=1.5$, $n_1=1$. In Fig. 4(b), the two DOM are shown for the same structure of Fig. 4(a), but with $n_h=2$, $n_1=1$. In Fig. 4(c), the two DOM are shown for the same structure as in Fig. 4(a), with an increased number of periods $N=8$ and $n_h=2$, $n_1=1$. We note the good agreement between the DOM predicted by the QNM theory and the DOM obtained by Bendickson.

V. CONCLUSION

We have analyzed the behavior of the electromagnetic field in the optical domain, inside one-dimensional photonic crystals, by using an extension of the QNM theory. 1D-PBG structures are particularly optical cavities, with both sides open to the external environment, with a stratified material inside. These PBG structures are finite in space, and when we work with electromagnetic pulses of a spatial extension longer than the length of the cavity, the PBG cannot be studied as an infinite structure, rather we have to consider the boundary conditions at the two ends of the structure.

The QNM theory, we have extended and applied, considers the realistic situation in which the cavity is open (from both sides) and is enclosed in an infinite external space. The

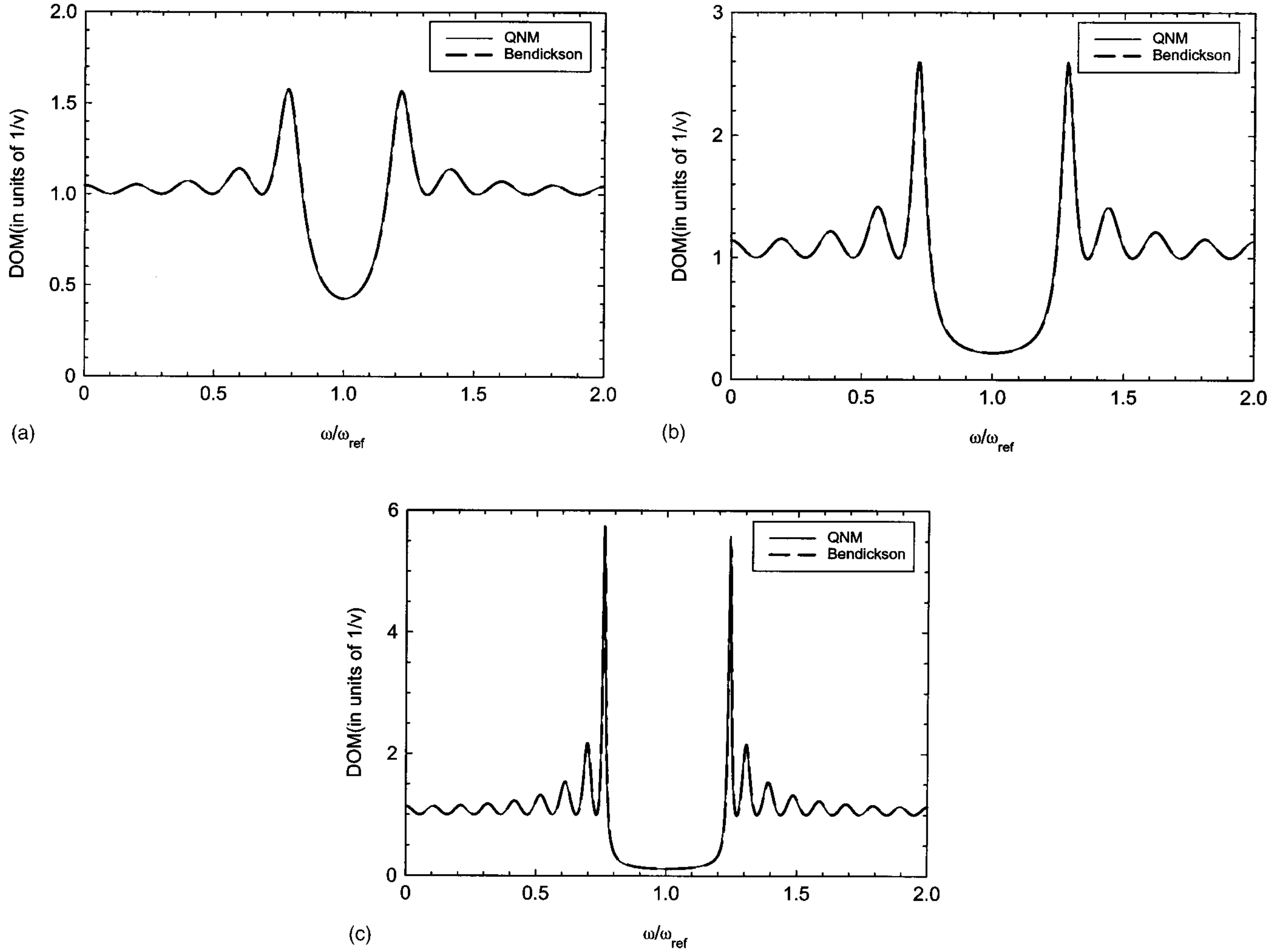

FIG. 4. (a) DOM in units of $1/v$, where $v = cL/L_{\mathrm{opt}}$ (L is the length and L_{opt} the optical path of the 1D PBG), according to the QNM theory (——) and Bendickson (– – –), for a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{\mathrm{ref}} = 1\ \mu\mathrm{m}$, number of periods $N = 4$, and refractive indices $n_h = 1.5$, $n_1 = 1$. (b) DOM in units of $1/v$, where $v = cL/L_{\mathrm{opt}}$ (L is the length and L_{opt} the optical path of the 1D PBG), according to the QNM theory (——) and Bendickson (– – –) for a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{\mathrm{ref}} = 1\ \mu\mathrm{m}$, number of periods $N = 4$, and refractive indices $n_h = 2$, $n_1 = 1$. (c) DOM in units of $1/v$, where $v = cL/L_{\mathrm{opt}}$ (L is the length and L_{opt} the optical path of the 1D PBG), according to the QNM theory (——) and Bendickson (– – –), for a symmetric quarter-wave 1D PBG with reference wavelength $\lambda_{\mathrm{ref}} = 1\ \mu\mathrm{m}$, number of periods $N = 8$, and refractive indices $n_h = 2$, $n_1 = 1$.

lack of energy conservation, for the system under consideration, gives complex, instead of real, eigenfrequencies. The evolution operator for the system, analogous of the Hamilton operator for the conservative cases, is not Hermitian for a double open optical cavity and the modes of the field are not normal but quasinormal.

Starting with the Green function we have discussed the completeness of the field representation inside the 1D-PBG structure. We have observed that the QNM frequencies are not uniformly distributed in the complex plane, but they arrange themselves in order to form permitted and forbidden bands, in agreement with the known characteristics of these structures [18]. Moreover, we have found that, for a symmetric quarter-wave 1D PBG with N periods and ω_{ref} as the reference frequency, there are exactly $2N+1$ QNM frequencies in the $[0,2\omega_{\mathrm{ref}}]$ range. One of them is located on the imaginary axis and we reject it because it does not represent a physical field oscillation, the remaining $2N$ QNM frequencies correspond to the $2N$ transmission peaks in the $[0,2\omega_{\mathrm{ref}}]$ range. Every QNM frequency characterizes the correspondent transmission peak because: (a) the real part of the QNM frequency is the resonance frequency of the transmission peak, and the imaginary part of QNM frequency is a measure of the broadening of the same peak; (b) the square modulus of the QNM function gives the field distribution in the 1D-PBG structure at the frequency of the peak. We have also demonstrated that the density of quasinormal modes can be given for a PBG structure.

The importance of the representation of the field in terms of QNMs lies in the fact that the properties of the fields in a fully open cavity are described; in particular, it is possible to recover the properties that are crucial for any other perturbative expansion of the field, as it is necessary to consider to treat nonlinear optical and quantum processes, as will be discussed in a forthcoming paper.

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APPENDIX A

In order to clarify our WKB-like approximation, we consider the function $g_+(x,\omega)$ [analogous considerations could be repeated for the function $g_-(x,\omega)$]. We want to solve the following differential equation:

$$\left(\partial_{xx}+\frac{n^2(x)\omega^2}{c^2}\right)g_+(x,\omega)=0 \tag{A1}$$

under the limit condition of $g_+(x,\omega)=e^{in_0(\omega/c)x}$ when $x\to\infty$.

We put $g_+(x,\omega)=e^{i(\omega/c)\xi(x)}$, then we obtain a similar differential equation for $\xi(x)$ function. Performing the double derivative of $g_+(x,\omega)$, we have

$$\partial_{xx}g_+=\partial_x\left(i\frac{\omega}{c}\frac{d\xi}{dx}g_+\right)=\left(i\frac{\omega}{c}\frac{d\xi}{dx}\right)^2 g_+ + i\frac{\omega}{c}\frac{d^2\xi}{dx^2}g_+ = -\frac{n^2(x)\omega^2}{c^2}g_+, \tag{A2}$$

i.e.,

$$\frac{\omega^2}{c^2}\left(\frac{d\xi}{dx}\right)^2-\frac{n^2(x)\omega^2}{c^2}=i\frac{\omega}{c}\frac{d^2\xi}{dx^2} \tag{A3}$$

and

$$\left(\frac{d\xi}{dx}\right)^2-n^2(x)=\frac{i}{2\pi}\lambda\frac{d^2\xi}{dx^2}. \tag{A4}$$

We are interested in solving the differential equation for g_+ in the limit of high values of ω, in order to prove the validity of Eq. (2.15). The WKB method, proposed to solve the Schrödinger equation (applied to the $\hbar$ parameter, considering $\hbar\to0$), here is applied in optics and for λ parameter (considering $\lambda\to0$)(see Ref. [9]). So, our WKB-like method starts from the hypothesis of expanding the derivative of $\xi(x)$ in powers of λ:

$$\frac{d\xi}{dx}=\varphi_0(x)+\lambda\varphi_1(x)+\lambda^2\varphi_2(x)+\cdots=\sum_n \lambda^n\varphi_n(x). \tag{A5}$$

Putting this expansion in the differential equation for $\xi(x)$, we have

$$\left(\sum_n \lambda^n\varphi_n(x)\right)^2-n^2(x)=\frac{i}{2\pi}\sum_n \lambda^{n+1}\frac{d\varphi_n(x)}{dx}. \tag{A6}$$

The first two orders for λ give the following recurrence relations:

$$\varphi_0^2(x)=n^2(x),$$

$$2\varphi_0(x)\varphi_1(x)=\frac{i}{2\pi}\frac{d}{dx}\varphi_0(x),$$

$$\varphi_1^2(x)+2\varphi_1(x)\varphi_0(x)=\frac{i}{2\pi}\frac{d}{dx}\varphi_1(x), \tag{A7}$$

and so on. From the first one, we have

$$\varphi_0(x)=\pm n(x) \tag{A8}$$

and from the second one,

$$\varphi_1(x)=\frac{i}{4\pi}\frac{1}{\varphi_0(x)}\frac{d}{dx}\varphi_0(x). \tag{A9}$$

If we stop our expansion to the zero order for λ, we obtain

$$d\xi(x)=\varphi_0(x)dx\Rightarrow\xi(x)=\pm\int_x^{x_j}n(x')dx' \tag{A10}$$

and the expression of $g_+(x,\omega)$ becomes

$$g_+(x,\omega)=\exp\left\{\pm i\frac{\omega}{c}\int_x^{x_1}n(x')dx'\right\}. \tag{A11}$$

This approximation is valid only if higher order can be neglected. The first-order term (in λ) can be neglected if

$$|\lambda\varphi_1(x)|\ll|\varphi_0(x)|, \tag{A12}$$

i.e., if it is much smaller than the zero-order term. Expressions found for $\varphi_0(x)e\varphi_1(x)$ give us

$$\left|\lambda\frac{i}{4\pi}\frac{1}{n(x)}\frac{\partial n(x)}{\partial x}\right|\ll|n(x)|, \tag{A13}$$

i.e.,

$$\left|\frac{dn(x)}{dx}\right|\frac{1}{|n^2(x)|}\ll\frac{4\pi}{\lambda}. \tag{A14}$$

In general, the refractive index *n value* is greater than 1, so the previous inequality can be fulfilled by the following one:

$$\left|\frac{dn(x)}{dx}\right|\ll\frac{4\pi}{\lambda}, \tag{A15}$$

that is, the final condition explaining the WKB-like method. Equation (2.14) is valid in the limit of $|\omega|\to\infty$ because in this case we have $\lambda\to0$. Equation (2.15) is also well defined for $|\omega|\to\infty$. With application of the WKB approximation we can obtain the exact solutions of the differential equation for g, for high values of frequencies, and we can use these relations in order to prove Eq. (2.14), i.e., the completeness of the QNM in open cavities [9].

APPENDIX B

In this appendix, we describe how to obtain the equation of QNM frequencies, Eqs. (3.3) and (3.4), for a symmetric 1D PBG with N periods, where a period consists of two layers with refractive indices n_h and n_l. Then, we solve this equation for a quarter-wave 1D PBG.

We use the matrix method [17] for describing the 1D-PBG structures. The transmission matrix for a single period of the 1D PBG has the form

$$M=\begin{pmatrix}\mu_{11} & \mu_{12}\\ \mu_{21} & \mu_{22}\end{pmatrix}, \tag{B1}$$

where

$$\mu_{11}=\cos\delta_h\cos\delta_l-\frac{q_h}{q_l}\sin\delta_h\sin\delta_l,$$
$$\mu_{12}=\frac{1}{q_h}\sin\delta_h\cos\delta_l+\frac{1}{q_l}\sin\delta_l\cos\delta_h,$$
$$\mu_{21}=-q_h\sin\delta_h\cos\delta_l-q_l\sin\delta_l\cos\delta_h,$$
$$\mu_{22}=\cos\delta_h\cos\delta_l-\frac{q_l}{q_h}\sin\delta_h\sin\delta_l. \tag{B2}$$

In expression (B2), the propagation constants in the two layers of a period $q_h=n_h(\omega/c)$, $q_l=n_l(\omega/c)$, and the respective phases $\delta_h=q_h h$, $\delta_l=q_l l$ appear.

The transmission matrix of a symmetric 1D PBG has the form

$$M_{\mathrm{PBG}}=\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}, \tag{B3}$$

where

$$m_{11}=\cos\delta_h[\mu_{11}U_{N-1}(\vartheta)-U_{N-2}(\vartheta)]\frac{\sin\delta_h}{q_h}+\mu_{21}U_{N-1}(\vartheta),$$
$$m_{12}=\cos\delta_h\mu_{12}U_{N-1}(\vartheta)+\frac{\sin\delta_h}{q_h}[\mu_{22}U_{N-1}(\vartheta)-U_{N-2}(\vartheta)],$$
$$m_{21}=-q_h\sin\delta_h[\mu_{11}U_{N-1}(\vartheta)-U_{N-2}(\vartheta)]+\cos\delta_h\mu_{21}U_{N-1}(\vartheta),$$
$$m_{22}=-q_h\sin\delta_h\mu_{12}U_{N-1}(\vartheta)+\cos\delta_h[\mu_{22}U_{N-1}-U_{N-2}(\vartheta)]. \tag{B4}$$

In expression (B4), the Chebyshev polynomials $U_N(\vartheta)=\sin[(N+1)\vartheta]/\sin\vartheta$ with argument $\cos\vartheta=(\mu_{11}+\mu_{22})/2$ appear.

As depicted in Fig. 2, if $f(x)$ is the wave function for a generic QNM and $g(x)=f'(x)$ is its spatial derivative, then (f_0,g_0) and (f_{2N+1},g_{2N+1}) are their values at the input and output of a 1D PBG, being

$$\begin{pmatrix}f_{2N+1}\\ g_{2N+1}\end{pmatrix}=M_{\mathrm{PBG}}\begin{pmatrix}f_0\\ g_0\end{pmatrix}. \tag{B5}$$

Referring to Fig. 1(b), if we solve in every layer the equation $[\partial_x^2+\rho(x)\omega^2]f(x)=0$, we obtain the expression for the wave function of a generic QNM in a symmetric 1D PBG:

$$\begin{aligned}f(x)=&(A_0e^{i(\omega/c)x}+B_0e^{-i(\omega/c)x})\vartheta(-x)\\&+\sum_{j=0}^{N}(A_{2j+1}e^{i(\omega/c)n_hx}+B_{2j+1}e^{-i(\omega/c)n_hx})\\&\times\vartheta[x-j(h+l)]\vartheta[j(h+l)]+\sum_{j=0}^{N-1}(A_{2j+2}e^{i(\omega/c)n_lx}\\&+B_{2j+2}e^{-i(\omega/c)n_lx})\vartheta[x-j(h+l)-h]\\&\times\vartheta[(j+1)(h+l)-x]+(A_{2N+2}e^{i(\omega/c)x}\\&+B_{2N+2}e^{-i(\omega/c)x})\vartheta[x-N(h+l)-h],\end{aligned} \tag{B6}$$

where $\vartheta(x)$ is the unitary step function.

If we impose the conditions for QNM resonances, $f(x)=\exp(\pm i\omega x/c)$ to $x\to\pm\infty$, we obtain $A_0(\omega)=0$ and

$$B_{2N+2}(\omega)=\frac{i(\omega/c)f_{2N+1}(\omega)-g_{2N+1}(\omega)}{2i(\omega/c)}e^{i(\omega/c)[N(h+l)+h]}=0. \tag{B7}$$

We can obtain Eqs. (3.3) and (3.4) after some algebra from Eq. (B7), if we insert Eqs. (B5), (B4), and (B2).

Now, if we work with a quarter-wave 1D PBG, we have

$$n_l l=n_h h=\frac{\lambda_{\mathrm{ref}}}{4}, \tag{B8}$$

where λ_{ref} is a reference wavelength. With these assumptions, the phases δ_h, δ_l become

$$\delta_l=\delta_h=\delta, \tag{B9}$$

where we have introduced the phase $\delta=(\lambda_{\mathrm{ref}}/4)(\omega/c)$. Equation (3.3) becomes a real coefficient polynomial equation (instead of a transcendental equation) of degree $2N+1$ in the variable $e^{i\delta}$. There are $2N+1$ families of QNMs and, if we pick a generic one, all the QNM frequencies have the same imaginary part and are distributed with a step $\Delta=2\omega_{\mathrm{ref}}$, where $\omega_{\mathrm{ref}}=2\pi c/\lambda_{\mathrm{ref}}$. Then, there are exactly $2N+1$ QNM frequencies in the $[0,2\omega_{\mathrm{ref}}]$ range.

From Eq. (B6), we can construct the QNM functions $f_n(x)$ for QNM frequencies $\omega=\omega_n$. We note that if we impose the conditions of continuity for the wave function $f(x)$ and its spatial derivative $g(x)=f'(x)$ at interfaces of 1D PBG, we obtain

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{-in_h(\omega/c)x_0} & \dfrac{1}{n_h} e^{-in_h(\omega/c)x_0} \\ e^{in_h(\omega/c)x_0} & \dfrac{-1}{n_h} e^{in_h(\omega/c)x_0} \end{pmatrix} \begin{pmatrix} e^{in_h(\omega/c)x_0} & e^{-in_h(\omega/c)x_0} \\ n_0 e^{in_h(\omega/c)x_0} & -n_0 e^{-in_h(\omega/c)x_0} \end{pmatrix} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix},$$

$$\begin{pmatrix} A_{2j} \\ B_{2j} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{-in_h(\omega/c)[x_0+(j-1)(h+1)+h]} & \dfrac{1}{n_l} e^{-in_h(\omega/c)[x_0+(j-1)(h+l)+h]} \\ e^{in_l(\omega/c)[x_0+(j-1)(h+l)+h]} & \dfrac{-1}{n_l} e^{in_l(\omega/c)[x_0+(j-1)(h+l)+h]} \end{pmatrix}$$

$$\times \begin{pmatrix} e^{in_h(\omega/c)[x_0+(j-1)(h+l)+h]} & e^{-in_h(\omega/c)[x_0+(j-1)(h+l)+h]} \\ n_h e^{in_h(\omega/c)[x_0+(j-1)(h+l)+h]} & -n_h e^{-in_h(\omega/c)[x_0+(j-1)(h+l)+h]} \end{pmatrix} \begin{pmatrix} A_{2j-1} \\ B_{2j-1} \end{pmatrix},$$

$$\begin{pmatrix} A_{2j+1} \\ B_{2j+1} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{-in_h(\omega/c)[x_0+j(h+l)]} & \dfrac{1}{n_h} e^{-in_h(\omega/c)[x_0+j(h+l)]} \\ e^{in_h(\omega/c)[x_0+j(h+l)]} & \dfrac{-1}{n_h} e^{in_h(\omega/c)[x_0+j(h+l)]} \end{pmatrix} \begin{pmatrix} e^{in_l(\omega/c)[x_0+j(h+l)]} & e^{-in_l(\omega/c)[x_0+j(h+l)]} \\ n_l e^{in_l(\omega/c)[x_0+j(h+l)]} & -n_1 e^{-in_l(\omega/c)[x_0+j(h+l)]} \end{pmatrix} \begin{pmatrix} A_{2j} \\ B_{2j} \end{pmatrix},$$

$$\begin{pmatrix} A_{2N+2} \\ B_{2N+2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{-in_h(\omega/c)[x_0+N(h+l)+h]} & \dfrac{1}{n_0} e^{-in_h(\omega/c)[x_0+N(h+l)+h]} \\ e^{in_h(\omega/c)[x_0+N(h+l)+h]} & \dfrac{-1}{n_0} e^{in_h(\omega/c)[x_0+N(h+l)+h]} \end{pmatrix}$$

$$\times \begin{pmatrix} e^{in_h(\omega/c)[x_0+N(h+l)+h]} & e^{-in_h(\omega/c)[x_0+N(h+l)+h]} \\ n_h e^{in_h(\omega/c)[x_0+N(h+l)+h]} & -n_h e^{-in_h(\omega/c)[x_0+N(h+l)+h]} \end{pmatrix} \begin{pmatrix} A_{2N+1} \\ B_{2N+1} \end{pmatrix}, \tag{B10}$$

where $A_0(\omega_n)=0$ and $B_{2N+1}(\omega_n)=0$.

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